

SM3 2.4: Synthetic Division and Remainder Thm. Key

Vocabulary Problems: Interpret the results of the synthetic division problem by rewriting the problem

as $\frac{\text{dividend}}{\text{divisor}} = \text{quotient}$.

$$1) \begin{array}{r|rrr} \boxed{2} & 3 & 5 & 7 \\ & \downarrow & 6 & 22 \\ \hline & 3 & 11 & \boxed{29} \end{array}$$

$$2) \begin{array}{r|rrr} \boxed{-7} & 1 & 2 & 4 \\ & \downarrow & -7 & -35 \\ \hline & 1 & -5 & \boxed{-31} \end{array}$$

$$\frac{3x^2 + 5x + 7}{x - 2} = 3x + 11 + \frac{29}{x - 2}$$

$$\frac{x^2 + 2x + 4}{x + 7} = x - 5 - \frac{31}{x + 7}$$

3) What is the remainder of $\frac{4x-11}{x-6}$?

$$\begin{array}{r|rr} \boxed{6} & 4 & -11 \\ & \downarrow & 24 \\ \hline & 4 & \boxed{13} \end{array}$$

4) What is the remainder of $\frac{5x^3-2x^2+4x-9}{x+3}$?

$$\begin{array}{r|rrrr} \boxed{-3} & 5 & -2 & 4 & -9 \\ & \downarrow & -15 & 51 & -165 \\ \hline & 5 & -17 & 55 & \boxed{-174} \end{array}$$

Problems: Simplify each expression into a quotient that contains a polynomial and $\frac{\text{remainder}}{\text{divisor}}$.

$$5) \frac{x^2 + 10x + 24}{x - 2}$$

$$\begin{array}{r|rrr} \boxed{2} & 1 & 10 & 24 \\ & \downarrow & 2 & 24 \\ \hline & 1 & 12 & \boxed{48} \end{array}$$

$$x + 12 + \frac{48}{x - 2}$$

$$6) \frac{x^2 - 3x + 7}{x - 4}$$

$$\begin{array}{r|rrr} \boxed{4} & 1 & -3 & 7 \\ & \downarrow & 4 & 4 \\ \hline & 1 & 1 & \boxed{11} \end{array}$$

$$x + 1 + \frac{11}{x - 4}$$

$$7) \frac{x^2 + 7x - 12}{x - 3}$$

$$\begin{array}{r|rrr} \boxed{3} & 1 & 7 & -12 \\ & \downarrow & 3 & 30 \\ \hline & 1 & 10 & \boxed{18} \end{array}$$

$$x + 10 + \frac{18}{x - 3}$$

$$8) \frac{x^2 + x - 30}{x + 6}$$

$$\begin{array}{r} \boxed{-6} \quad 1 \quad 1 \quad -30 \\ \downarrow \quad -6 \quad 30 \\ \hline 1 \quad -5 \quad \boxed{0} \end{array}$$

$$x - 5$$

$$9) \frac{x^2 + 11x + 8}{x + 1}$$

$$\begin{array}{r} \boxed{-1} \quad 1 \quad 11 \quad 8 \\ \downarrow \quad -1 \quad -10 \\ \hline 1 \quad 10 \quad \boxed{-2} \end{array}$$

$$x + 10 + \frac{-2}{x + 1}$$

$$10) \frac{x^2 + 4x - 5}{x}$$

$$\begin{array}{r} \boxed{0} \quad 1 \quad 4 \quad -5 \\ \downarrow \quad 0 \quad 0 \\ \hline 1 \quad 4 \quad \boxed{-5} \end{array}$$

$$x + 4 + \frac{-5}{x}$$

$$11) \frac{3x^2 + x + 6}{x - 8}$$

$$\begin{array}{r} \boxed{8} \quad 3 \quad 1 \quad 6 \\ \downarrow \quad 24 \quad 200 \\ \hline 3 \quad 25 \quad \boxed{206} \end{array}$$

$$3x + 25 + \frac{206}{x - 8}$$

$$12) \frac{-4x^2 - 3x - 1}{x + 6}$$

$$\begin{array}{r} \boxed{-6} \quad -4 \quad -3 \quad -1 \\ \downarrow \quad 24 \quad -126 \\ \hline -4 \quad 21 \quad \boxed{-127} \end{array}$$

$$-4x + 21 + \frac{-127}{x + 6}$$

$$13) \frac{2x^2 - 12}{x - 5}$$

$$\begin{array}{r} \boxed{5} \quad 2 \quad 0 \quad -12 \\ \downarrow \quad 10 \quad 50 \\ \hline 2 \quad 10 \quad \boxed{38} \end{array}$$

$$2x + 10 + \frac{38}{x - 5}$$

$$14) \frac{x^3 - 5x^2 + 10x - 50}{x - 5}$$

$$\begin{array}{r} \boxed{5} \quad 1 \quad -5 \quad 10 \quad -50 \\ \downarrow \quad 5 \quad 0 \quad 50 \\ \hline 1 \quad 0 \quad 10 \quad \boxed{0} \end{array}$$

$$x^2 + 10$$

$$15) \frac{x^3 + x^2 + x + 1}{x - 1}$$

$$\begin{array}{r} \boxed{1} \quad 1 \quad 1 \quad 1 \quad 1 \\ \downarrow \quad 1 \quad 2 \quad 3 \\ \hline 1 \quad 2 \quad 3 \quad \boxed{4} \end{array}$$

$$x^2 + 2x + 3 + \frac{4}{x - 1}$$

$$16) \frac{2x^3 + 3x^2 + 4x - 5}{x + 3}$$

$$\begin{array}{r} \boxed{-3} \quad 2 \quad 3 \quad 4 \quad -5 \\ \downarrow \quad -6 \quad 9 \quad -39 \\ \hline 2 \quad -3 \quad 13 \quad \boxed{-44} \end{array}$$

$$2x^2 - 3x + 13 + \frac{-44}{x + 3}$$

$$17) \frac{x^5 - 4x^4 - 2x^3 - 6x^2 + 9x + 3}{x - 2}$$

$$\begin{array}{r} \boxed{2} \quad 1 \quad -4 \quad -2 \quad -6 \quad 9 \quad 3 \\ \downarrow \quad 2 \quad -4 \quad -12 \quad -36 \quad -54 \\ \hline 1 \quad -2 \quad -6 \quad -18 \quad -27 \quad -51 \end{array}$$

$$x^4 - 2x^3 - 6x^2 - 18x - 27 + \frac{-51}{x - 2}$$

$$18) \frac{7 - x^3 - 3x^4 + x^2 - 3x^5}{x + 3}$$

$$\begin{array}{r} \boxed{-3} \quad -3 \quad -3 \quad -1 \quad 1 \quad 0 \quad 7 \\ \downarrow \quad 9 \quad -18 \quad 57 \quad -174 \quad 522 \\ \hline -3 \quad 6 \quad -19 \quad 58 \quad -174 \quad 529 \end{array}$$

$$-3x^4 + 6x^3 - 19x^2 + 58x - 174 + \frac{529}{x + 3}$$

19) Prove whether $(x + 5)$ is a factor of $x^2 + 2x - 35$ and write a sentence explaining your reasoning.

$$\begin{array}{r} \boxed{-5} \quad 1 \quad 2 \quad -35 \\ \downarrow \quad -5 \quad -15 \\ \hline 1 \quad -3 \quad \boxed{-50} \end{array}$$

Because the remainder of $\frac{x^2+2x-35}{x+5}$ is not 0, $x + 5$ is not a factor of $x^2 + 2x - 35$.

20) Prove whether $(x - 2)$ is a factor of $x^3 - 3x^2 + 4$ and write a sentence explaining your reasoning.

$$\begin{array}{r} \boxed{2} \quad 1 \quad -3 \quad 0 \quad 4 \\ \downarrow \quad 2 \quad -2 \quad -4 \\ \hline 1 \quad -1 \quad -2 \quad \boxed{0} \end{array}$$

Because the remainder of $\frac{x^3-3x^2+4}{x-2}$ is 0, $x - 2$ is a factor of $x^3 - 3x^2 + 4$.

21) Given $f(x) = 4x^5 - 2x^3 + 17x^2 + 4$, find $f(-3)$.

$$\begin{array}{r} \boxed{-3} \quad 4 \quad 0 \quad -2 \quad 17 \quad 0 \quad 4 \\ \downarrow \quad -12 \quad 36 \quad -102 \quad 255 \quad -765 \\ \hline 4 \quad -12 \quad 34 \quad -85 \quad 255 \quad \boxed{-761} \end{array}$$

Since the remainder is -761 , then $f(-3) = -761$.

22) Given $f(x) = -7x^3 + 2x^2 + 15x - 26$, find $f(13)$.

$$\begin{array}{r} \boxed{13} \quad -7 \quad 2 \quad 15 \quad -26 \\ \downarrow \quad -91 \quad -1157 \quad -14846 \\ \hline -7 \quad -89 \quad -1142 \quad \boxed{-14872} \end{array}$$

$$f(13) = -14872$$